DIFFERENTIAL DYNAMIC PROGRAMMING FOR ESTUARINE MANAGEMENT

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ABSTRACT: A differential dynamic programming (DDP) procedure is applied to solve both linear and nonlinear estuarine-management problems to determine the optimal amount of freshwater inflows into bays and estuaries to maximize fishery harvests. Fishery harvests are expressed in regression equations as functions of freshwater inflows. The optimization problem is posed as a discrete-time optimal control problem in which salinity represents the state variable and freshwater inflow represents the control variable. Both linear and nonlinear regression equations that relate salinity to freshwater inflow are used as the transition equations. The bound constraints for the control and state variables are incorporated into the objective function using a penalty-function method to convert the problem into an unconstrained formulation. To guarantee the quadratic convergence of the DDP procedure, a constant-shift and an adaptive-shift method are used. The DDP procedure is applied to the Lavaca-Tres Palacios estuary in Texas and the results are compared with a nonlinear programming solver. This work demonstrates the potential of DDP for developing a more complex model that uses a two-dimensional hydrodynamic-salinity transport model for the transition.

INTRODUCTION

In many areas of the United States, particularly the Gulf coast states and California, estuaries are important natural resources because they provide areas of nursery habitats for juvenile forms of marine species, for sport and commercial fishing, and for other recreational activities.

The ecosystem of an estuary is largely dependent on the amount, as well as the seasonal and spatial distribution of freshwater inflows and the associated nutrients. Freshwater inflows enter the estuary from rivers and streams and from local rainfall runoff. Freshwater dilutes the saline tidal water seaward and transports nutrient and sediment that maintain marsh environments and contribute to the estuarine production of fish and shellfish.

An estuarine system is complicated because of the interaction of many physical, chemical, and biological parameters. Among the parameters influencing estuarine productivity, many are beyond our control, such as wind and temperature. Estuarine management is a means to maintain the estuary system in a desired condition by adjusting the controllable parameters. Since freshwater is one of the most important parameters to the estuary system, in essence, estuarine management is to manage freshwater resources in order to provide an optimal estuarine environment. The specific action of estuarine management can be in several forms, such as minimizing the total volume of freshwater into an estuary, maximizing the upstream water uses, or maximizing the commercial fishery harvest.

In this paper, the mathematical programming model for estuarine management is reformulated as a discrete-time optimal control problem. The differential dynamic programming (DDP) method is used to solve the problem. The objective function is the enhancement of fishery harvest, i.e., to optimize the freshwater inflow into bays and estuaries to maximize the total annual commercial harvest of selected fish species while meeting the viability limits for salinity, and satisfying monthly and seasonal freshwater inflow needs. The model is applied to the Lavaca-Tres Palacios Estuary in Texas (see Fig. 1). The computer model developed using the DDP method can provide a useful tool for decision makers to quantitatively analyze water-management strategies. The major purpose of this paper is to demonstrate the potential of the DDP for developing a more complex model that uses a two-dimensional hydrodynamic-salinity transport model for the transition.

ESTUARINE MANAGEMENT

The estuarine condition results from the interaction of salinity, nutrients, and key organisms with factors such as tide, wind, precipitation, evaporation, and some unique conditions associated with the specific estuary. The primary indicators are salinity and nutrients. Salinity is an index, which has been well established to indicate ecological conditions in an estuary, because it can measure the relative proportion of freshwater to sea water. The lower and upper salinity bounds for a specific organism are set based on either the presence of that organism in the estuary as reflected in the catch data and corresponding salinity value, or on the salinity physiological dependence for viable metabolic and reproductive activity as revealed by laboratory studies. The mathematical relationship between salinity and freshwater inflow in the estuary is developed based on statistical association, i.e., a regression form established from field data by the Texas Water Development Board (Texas 1980).

In the estuaries of Texas, a direct measure of organism

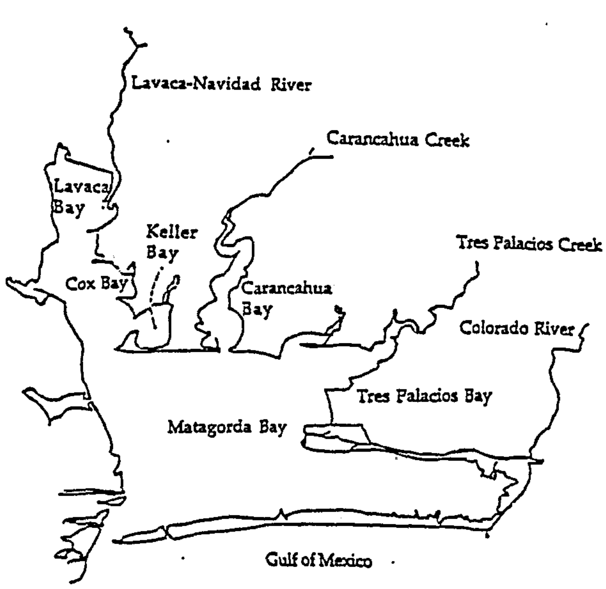

FIG. 1. Lavaca-Tres Palacios Estuary in Texas

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TABLE 1. Regression Equations of Fishery Harvest and Freshwater Inflow Relation (Texas 1980)

Spices (1)	Equations (2)	Inflow used in regression equation (3)
All shellfish	$H_1 = 3107.9 - 11.3QS_1$[a] $+ 7.7QS_2$[b] $- 24.2QS_3$[c]	Freshwater inflow at Lavaca Bay
All penaeid shrimp	$H_2 = 1735.8 - 3.7QS_1 + 2.7QS_2 - 1.0QS_5$	Combined freshwater inflows from all contributing rivers and coastal drainage basins
Blue crab	$H_3 = 208.3 + 2.7QS_3 +$ $0.4QS_4$[d] $+ 0.5QS_5$[e]	Combined freshwater inflows from all contributing rivers and coastal drainage basins

[a]January–March.
[b]April–June.
[c]July–August.
[d]September–October.
[e]November–December.

abundance is available on the data of commercial fishery landings taken from the estuary. This fishery harvest data can be employed as an index to populations of key organisms and can be statistically analyzed to establish its dependence on freshwater inflow, $H_k = f(Q)$. The regression equations of fishery harvest and freshwater inflow are listed in Table 1 and were developed by the Texas Water Development Board (Texas 1980). [In Table 1, H_k is the commercial harvest in thousands of pounds (1 lb = 0.45 kg), and QS_j is the mean monthly freshwater inflow during the season in 1,000 acre-ft (1 acre-ft = 1,233.5 m^3).]

The estuarine management problem can be formulated with different management objectives or even as a multiobjective problem. One of the objectives is the enhancement of fishery harvest of selected fish species while meeting the viability limits for salinity and satisfying monthly and seasonal freshwater inflow needs. The mathematical model can be expressed as follows:

$$\max \sum H_k \tag{1}$$

$$S_{t,j} = \beta_0 Q_{t,j}^{\beta_1} \tag{2}$$

$$\underline{Q}_{t,j} \le Q_{t,j} \le \bar{Q}_{t,j} \tag{3}$$

$$\underline{S}_{t,j} \le S_{t,j} \le \bar{S}_{t,j} \tag{4}$$

$$\underline{QS}_{j,n} \le QS_{j,n} \le \overline{QS}_{j,n} \tag{5}$$

where H_k = fishery harvest for the kth species (1,000 lb or 1,000 kg); $Q_{t,j}$ = tth monthly inflows from the jth river (cfs or m^3/s); $S_{t,j}$ = tth monthly average salinity at a specified location in the estuary, for river j (ppt); β_0 and β_1 = coefficients; $\bar{S}_{t,j}$ and $\underline{S}_{t,j}$ = upper and lower limits on monthly average salinity at a specified location in the estuary, for river j (ppt); $\bar{Q}_{t,j}$ and $\underline{Q}_{t,j}$ = upper and lower limits of monthly freshwater inflow (cfs or m^3/s); $QS_{j,n}$ = mean monthly flow in season n from river j (cfs or m^3/s), where $QS_{j,n} = (1/N_n) \Sigma_{t \in M_n} Q_{t,j}$, M_n is the set of months in season n and N_n is the number of months in season n; and $\overline{QS}_{j,n}$ and $\underline{QS}_{j,n}$ = upper and lower limits, respectively, on the mean monthly freshwater inflow in season n from river j (cfs or m^3/s).

Eq. (2) defines the relation of the state variable (salinity) and the control variable (freshwater inflow) at time t. Constraint (3) defines bounds on the monthly inflows. Constraint (4) defines bounds on the monthly salinity, and constraint (5) defines bounds on the monthly inflow during the season. Alternatively, the objective function (1) could be substituted to minimize freshwater inflow with the same constraint set.

Martin (1987) was the first to formulate this problem as an optimization problem and solved it using linear programming. This problem was also solved as a chance constrained nonlinear programming problem by Tung et al. (1990) using GRG2 (Lasdon and Waren 1986). Bao (1992) and Bao and Mays (1994) solved the estuarine problem as a discrete-time optimal control problem using a mathematical programming approach interfacing GRG2 with the two-dimensional hydrodynamic-salinity transport model HYD-SAL (Texas 1980). Mao and Mays (1994) formulated the estuarine-management problem as a multiobjective goal programming problem and solved it using the nonlinear solvers, GAMS (Brooke et al. 1988) and GRG2. Other multiobjective models for the estuarine-management problem were developed by Shi (1992), LeBlanc (1993), and Siebert (1993).

In this paper, the DDP method is applied to the Lavaca-Tres Palacios Estuary in Texas, shown in Fig. 1. The magnitude of freshwater inflow is one of the most important factors controlling the changes in estuarine salinity patterns. The main freshwater inflow sources considered are the Colorado River, which principally influences Matagorda Bay, and the Lavaca River, which principally influences Lavaca Bay. The freshwater inflow in this estuary is controlled by releases from the upstream reservoir of the Highland Lake System in the Colorado River Basin and Lake Texana in the Lavaca River Basin. The main advantage of the DDP method is that no discretization of the control and state space is used as compared with dynamic programming. The computational effort to solve for a nonsteady control policy increases only linearly with the number of time steps N. In contrast, when using a nonlinear programming algorithm, the computational effort to solve for a control policy that changes with time would increase rapidly, typically N^R, where $2 < R < 3$ (Culver and Shoemaker 1992). Therefore, the DDP method may have an advantage over mathematical programming approaches, especially when the equality constraint is a complex simulation model.

ALGORITHM DESCRIPTION

DDP algorithms have been used in water-resources applications such as reservoir operation and ground-water–management problems (Chang et al. 1992; Culver and Shoemaker 1993). Unconstrained discrete-time optimal control problems have the following general form:

$$\min J = \sum_{t=1}^{N} g(\mathbf{x}_t, \mathbf{u}_t, t) \tag{6}$$

$$\text{subject to } \mathbf{x}_{t+1} = T(\mathbf{x}_t, \mathbf{u}_t, t) \tag{7}$$

$t = 1, 2, \ldots, N-1, N$, $\mathbf{x}_1 = \mathbf{x}_1^c$ is given and fixed, where $\mathbf{x}_t$ = state variable; $\mathbf{u}_t$ = control variable; $g(\mathbf{x}_t, \mathbf{u}_t, t)$ = loss function; and $T(\mathbf{x}_t, \mathbf{u}_t, t)$ = transition function.

The DDP algorithm is an iterative algorithm in which, at each iteration, there are two sweeps. (1) A "backward sweep" to compute a series of coefficients following the dynamic programming optimal scheme; and (2) a "forward sweep" to update the sequences of state and control variables $(\mathbf{x}_t, \mathbf{u}_t)$ through the transition equation and the feedback function $\mathbf{u}_t = \alpha_t + \beta_t(\mathbf{x}_t - \mathbf{x}_t^c) + \mathbf{u}_t^c$ forward in time. An algorithm description of the DDP algorithm was given by Yakowitz and Rutherford (1984).

The practical optimal control problems often include constraints that can be expressed as

$$L(\mathbf{x}_t, \mathbf{u}_t, t) \ge 0 \tag{8}$$

Several techniques can be used to handle the constraints (Jones et al. 1987; Yakowitz 1989; Andricevic and Kitanidis 1990; Chang et al. 1992). One technique is through the use of a penalty function in which the penalty function $P(\mathbf{x}_t, \mathbf{u}_t, t, R)$ is associated with the violation of constraints and is added to the objective function. There are many kinds of penalty functions that can be used. Some penalty functions are complicated, and include the estimation of many parameters. Here, a very simple penalty function, a bracket penalty function (Reklaitis et al. 1983), is selected in the algorithm description and model formulation. In the application, the Lagrangian penalty function (Luenberger 1984) and the hyperbolic penalty functions (Lin 1990) are also used to compare the effect of different penalty functions.

The bracket function has the following form:

$$P(\mathbf{x}_t, \mathbf{u}_t, t, R) = R\cdot[L(\mathbf{x}_t, \mathbf{u}_t, t)]^2 \tag{9}$$

where R = a penalty parameter, which is a numerical value that must be assigned; and $\langle L(\mathbf{x}_t, \mathbf{u}_t, t)\rangle$ = constraint violation.

$$[L(\mathbf{x}_t, \mathbf{u}_t, t)] = 0, \quad \text{if} \quad L(\mathbf{x}_t, \mathbf{u}_t, t) \geq 0 \tag{10a}$$

$$[L(\mathbf{x}_t, \mathbf{u}_t, t)] = L(\mathbf{x}_t, \mathbf{u}_t, t), \quad \text{if} \quad L(\mathbf{x}_t, \mathbf{u}_t, t) < 0 \tag{10b}$$

The penalty function is added to equation (6) so that the augmented objective function becomes

$$\min \hat{J} = \sum_{t=1}^{N} \hat{G}(\mathbf{x}_t, \mathbf{u}_t, t, R) \tag{11}$$

where

$$\hat{G}(\mathbf{x}_t, \mathbf{u}_t, t, R) = g(\mathbf{x}_t, \mathbf{u}_t, t) + P(\mathbf{x}_t, \mathbf{u}_t, t, R)$$

The step-search technique of the DDP method by Yakowitz and Rutherford (1984) for unconstrained problems is now briefly described. ε is defined as a positive number, and the policy $\mathbf{u}(\varepsilon)$, associated with the components (α_t, β_t), is determined by the following recursive formulas applied for $t = 1, 2, \ldots, N$

$$\mathbf{u}_t(\varepsilon) = \mathbf{u}_t^c + \varepsilon\alpha_t + \beta_t(\mathbf{x}_t - \mathbf{x}_t^c) \tag{12}$$

$$\mathbf{x}_{t+1} = T[\mathbf{x}_t, \mathbf{u}_t(\varepsilon), t] \tag{13}$$

The procedure initially sets $\varepsilon = 1$ and defines θ_1 as

$$\theta_1 = \frac{1}{2}\sum_{t=1}^{N} - \mathbf{D}_t^T\mathbf{C}_t^{-1}\mathbf{D}_t \tag{14}$$

where $\mathbf{C}_t$, $\mathbf{D}_t$, α_t, and β_t = matrices of coefficients defined by the current control policy and trajectory. If the following relationship is satisfied:

$$\{J[\mathbf{u}(\varepsilon)] - J(\mathbf{u}^c)\} < \frac{1}{2}\theta_1\varepsilon \tag{15}$$

then $\mathbf{u}(\varepsilon)$ is accepted as the successor policy and is used in the next DDP iteration. Otherwise, ε is redefined to be one-half its former value and the policy $\mathbf{u}(\varepsilon)$ and the test defined by (15) are again computed. This process of halving ε and computing and testing $\mathbf{u}(\varepsilon)$ continues until acceptance occurs. But for the constrained problem, this step-search technique of finding the optimal policy needs to be modified. When the control or state variable "step outside" the constrained region (Murray and Yakowitz 1979), the positive penalty terms are added to the objective function. The successor policy $\mathbf{u}(\varepsilon)$ may result in a worse objective function value than the current control policy $\mathbf{u}^c$. If using (12) and (13) $\mathbf{u}(\varepsilon)$ converges to such a nonoptimal policy, then the step-search technique fails for the constrained problem, especially when the initial policy is far from an optimal policy. Only when an initial guess is sufficiently close to an optimal policy can the optimal control policy be reached. Thus, much work must be performed to find a suitable initial policy so that the sequence $\mathbf{u}(\varepsilon)$ converges to an optimal policy. To overcome this problem, the step-search technique of the unconstrained DDP method is modified as follows:

Step 1

Give the inner optimal convergence criterion ε_1, the overall convergence criterion θ_{min}, and the bound violation criteria, modify (12) and (13) as follows:

$$\mathbf{u}_t = \alpha_t + \beta_t(\mathbf{x}_t - \mathbf{x}_t^c) + \mathbf{u}_t^c \tag{16}$$

$$\mathbf{u}_{t+1} = T(\mathbf{x}_t, \mathbf{u}_t, t) \tag{17}$$

The test (15) is modified as

$$\max_t |\mathbf{u}_t - \mathbf{u}_t^c| \leq \varepsilon_1 \tag{18}$$

If (18) is satisfied, the DDP inner optimal is reached. If (18) is not satisfied, update the current control policy and the test (18) is again computed until the inner optimal is reached.

Step 2

If $|\theta_1| \leq \theta_{min}$ [θ_1 is defined by (14)], the overall optimal is reached. Otherwise, check whether the bound violations satisfy the given criteria. If the criteria are satisfied, keep the same penalty parameters. otherwise update the penalty parameters and repeat the iteration process until an overall optimal is reached.

Computational results show that the modified step-search technique can make the constrained DDP procedure converge to an optimal control policy. The flowchart of the modified DDP method is shown in Fig. 2.

MODEL FORMULATION

To formulate the optimal control problem, the objective function must be separable. In the estuarine-management model, the objective function is to maximize the summation of fishery harvest. Fishery harvest is expressed in terms of mean monthly freshwater inflow per season, as illustrated in Table 1. Three fish species are considered for the objective, i.e., all shellfish, all penaeid shrimp, and blue crab. These regression equations were developed by the Texas Department of Water Resources (1980). A separable objective function can be obtained by replacing the monthly freshwater inflow in each season with monthly freshwater inflows as

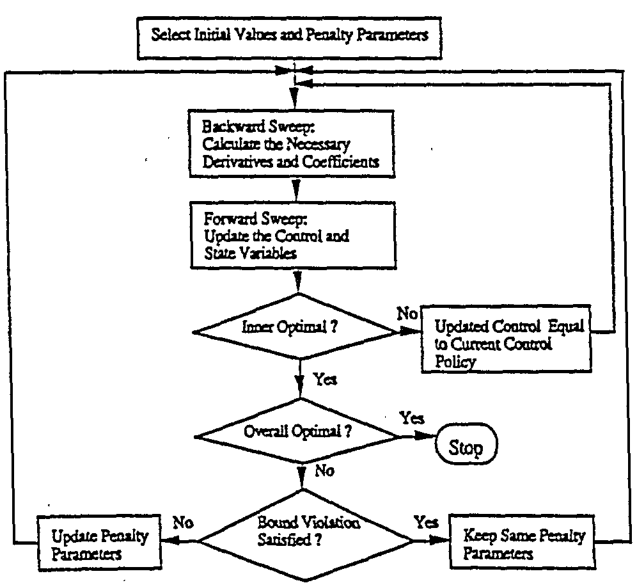

FIG. 2. Flowchart of Modified DDP Method

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$$\max J = \sum_{t=1}^{N} \sum_{j=1}^{m} g(Q_{t,j}, t) \quad (19)$$

where $g(Q_{t,j}, t) = (a_{t,j} + b_{t,j}Q_{t,j})$ = regression equations expressed in terms of freshwater inflow; $a_{t,j}$ and $b_{t,j}$ = coefficients that are given in Table 2; N = time period; and m = dimension of the control variable. It is convenient to transform the maximization into a minimization; thus, the objective function for the estuarine-management problem becomes

$$\min \bar{J} = -\sum_{t=1}^{N} \sum_{j=1}^{m} g(Q_{t,j}, t) \quad (20)$$

The DDP method requires the state transition equation to represent the relationship of the state variable at time t and at time $t + 1$. In the estuarine-management problem, salinity is chosen as the state variable, and monthly freshwater inflow is chosen as the control variable. For the sake of illustration, both linear and nonlinear regression equations serve as the state transition equations for the DDP model. The transition equations were derived from historical data (Texas 1980). Both linear and nonlinear state transition equations of the following format were used:

$$S_{t+1,j} = a_j' + b_j'S_{t,j} + c_j'Q_{t,j} \quad (21)$$

$$S_{t+1,j} = a_j'' + b_j''S_{t,j} + c_j''Q_{t,j} + d_j''S_{t,j}Q_{t,j} + e_j''S_{t,j}^2 + f_j''Q_{t,j}^2 \quad (22)$$

where a_j', b_j', c_j', a_j'', b_j'', c_j'', d_j'', e_j'', and f_j'' = coefficients that are determined by the least-squares method and are listed in Tables 3 and 4; $S_{t,j}$ and $S_{t+1,j}$ = salinities (ppt) corresponding to the beginning of months t and $t + 1$ for river j; and $Q_{t,j}$ = t monthly freshwater inflow (cfs or m³/s) from river j.

The bound constraints on monthly freshwater inflow and the bound constraints on monthly freshwater inflow in each season can be combined together as the bound constraints of the control variable (freshwater inflow)

$$\max(\underline{Q}_{t,j}, \underline{QS}_{j,n}) \le Q_{t,j} \le \min(\bar{Q}_{t,j}, \overline{QS}_{j,n}) \quad (23)$$

where $\underline{QS}_{j,n}$ and $\overline{QS}_{j,n}$ = lower and upper bounds of mean monthly freshwater inflow in season n, respectively. Let $\underline{Q}'_{t,j} = \max(\underline{Q}_{t,j}, \underline{QS}_{j,n})$ and $\bar{Q}'_{t,j} = \min(\bar{Q}_{t,j}, \overline{QS}_{j,n})$, then the combined bound constraints on monthly freshwater inflows are

$$\underline{Q}'_{t,j} \le Q_{t,j} \le \bar{Q}'_{t,j} \quad (24)$$

The values of combined bound constraints for monthly freshwater inflow are listed in Table 5.

Using the bracket penalty function, the penalty term associated with combined bound constraints on control variables is

$$P_1 = R_1 \cdot (\min\{0, \min[(Q_{t,j} - \underline{Q}'_{t,j}), (\bar{Q}'_{t,j} - Q_{t,j})]\})^2 \quad (25)$$

According to the DDP algorithm, it is known that $S_{t+1,j}$ is determined by $S_{t,j}$ and $Q_{t,j}$, and that meanwhile $S_{t+1,t}$ should also satisfy its bound constraint, i.e., $\underline{S}_{t+1,j} \le S_{t+1,j} \le \bar{S}_{t+1,j}$. This means that the control variable $Q_{t,j}$ is determined such that $S_{t+1,j}$ is also within its bounds. The value of the salinity bound constraints are listed in Table 6. The penalty term associated with the bound constraints of the state variables is

$$P_2 = R_2 \cdot (\min\{0, \min[(S_{t+1,j} - \underline{S}_{t+1,j}), (\bar{S}_{t+1,j} - S_{t+1,j})]\})^2 \quad (26)$$

After introducing the penalty function into the objective function the augmented optimal control model is represented as

$$\min \hat{J} = \sum_{t=1}^{N} \sum_{j=1}^{m} \hat{G}(Q_{t,j}, S_{t,j}, t, R_1, R_2) \quad (27)$$

$$\text{subject to } S_{t+1,j} = T(S_{t,j}, Q_{t,j}, t) \quad (28)$$

where $\hat{G}(Q_{t,j}, S_{t,j}, t, R_1, R_2) = -g(Q_{t,j}, t) + P_1 + P_2$; and P_1 and P_2 = defined by (25) and (26), respectively. The transition (28) is defined by (21) or (22) for linear or nonlinear transiton equation, respectively, for the example application presented here. Other forms of nonlinear transition equation could also be used.

TABLE 2. Coefficients $a_{t,j}$ and $b_{t,j}$ in Objective Function

Month (1)	$a_{t,j}$ Lavaca Bay (2)	$a_{t,j}$ Matagorda Bay (3)	$b_{t,j}$ Lavaca Bay (4)	$b_{t,j}$ Matagorda Bay (5)
January	463.06	74.57	−0.307	−0.076
February	455.05	66.56	−0.278	−0.068
March	464.3	75.81	−0.307	−0.076
April	531.18	142.69	0.206	0.054
May	559.98	171.49	0.213	0.055
June	552.33	163.84	0.206	0.054
July	427.45	38.96	−0.661	0.083
August	427.9	39.41	−0.661	0.083
September	32.46	32.46	0.012	0.012
October	31.66	31.66	0.012	0.012
November	120.97	120.97	−0.015	−0.015
December	118.6	118.6	−0.015	−0.015

TABLE 3. Coefficients of Linear Transition Equations

Rivers (1)	Coefficients a_j' (2)	b_j' (3)	c_j' (4)	Sum of squares of residual (5)
Colorado River	1.323	0.611	0.002	22.2111
Lavaca River	4.751	0.641	0.001	23.1823

CONVERGENCE OF DDP PROCEDURE

To guarantee the quadratic convergence of the DDP procedure, it is required that the stage-wise Hessian matrices $\mathbf{C}_t$ computed in the algorithm are positive-definite; otherwise the DDP procedure may not converge. Two methods are employed to obtain the positive definite Hessian matrices $\mathbf{C}_t$ in this paper. A constant-shift method and an adaptive-shift method. The constant shift method is as follows:

TABLE 4. Coefficients of Nonlinear Transition Equations

Rivers (1)	Coefficients a_j'' (2)	b_j'' (3)	c_j'' (4)	d_j'' (5)	e_j'' (6)	f_j'' (7)	Sum of squares of residual (8)
Colorado River	0.0531	3.0014	−0.0147	0.0007	−0.1539	0.0000029	12.6572
Lavaca River	37.7043	−1.7045	0.0024	−0.0014	0.1004	0.0000057	6.9212

TABLE 5. Combined Bounds of Freshwater Inflow

	Lavaca River		Colorado River	
Month (1)	Lower bounds (cfs) (2)	Upper bounds (cfs) (3)	Lower bounds (cfs) (4)	Upper bounds (cfs) (5)
January	37	1,903	293	3,009
February	41	2,107	324	3,331
March	37	1,903	293	7,500
April	62	6,400	392	7,600
May	60	7,300	379	8,863
June	62	8,286	392	9,209
July	114	732	276	6,017
August	114	732	276	2,900
September	294	3,272	387	5,024
October	285	3,166	374	4,879
November	114	1,739	573	7,865
December	111	1,683	496	6,200

Note: 1 cfs = 0.0283 m³/s.

TABLE 6. Salinity Bounds of Upper Lavaca Bay and Eastern Arm of Matagorda Bay

	Lavaca Bay (Lavaca River)		Matagorda Bay (Colorado River)	
Month (1)	Lower bounds (ppt) (2)	Upper bounds (ppt) (3)	Lower bounds (ppt) (4)	Upper bounds (ppt) (5)
January	10	20	10	30
February	10	20	10	30
March	10	20	10	25
April	5	15	5	20
May	1	15	5	20
June	1	15	5	20
July	10	20	10	25
August	10	20	10	25
September	5	15	5	20
October	5	15	5	20
November	10	20	10	30
December	10	20	10	30

Note: Principal river shown in parentheses.

Pick a constant $\varepsilon^c > 0$ (ε^c is independent of t) such that $\mathbf{C}_t^c$ is positive-definite where $\mathbf{C}_t^c$ is defined as

$$\mathbf{C}_t^c = \mathbf{C}_t + \varepsilon^c \mathbf{I}_m \tag{29}$$

where ε^c = a positive number; and $\mathbf{I}_m$ = identity.

The adaptive-shift method suggested by Liao and Shoemaker (1991) to obtain positive-definite Hessian matrices C_t was used. The main steps are as follows:

1. Adopt the constant-shift method
2. For a given constant $\delta > 0$, define $\mathbf{C}_t^a$ as follows:

$$\mathbf{C}_t^a = \mathbf{C}_t + \varepsilon_t^a(\delta)\mathbf{I}_m \tag{30}$$

where

$$\varepsilon_t^a = \begin{cases} \delta - \lambda(\mathbf{C}_t), & \text{if } \lambda(\mathbf{C}_t) < \delta \\ 0, & \text{if } \lambda(\mathbf{C}_t) \ge \delta \end{cases} \tag{31}$$

$\lambda(\mathbf{C}_t)$ = minimum eigenvalue of $\mathbf{C}_t$.

The adaptive-shift method consists of a constant shift procedure (step 1) and an active shift procedure (step 2). For a linear transition equation, the convergence speed of using the adaptive-shift method is faster than that of using the constant-shift method. For a nonlinear transition equation, computational results show that use of the adaptive-shift method does not always result in *positive-definite Hessian matrices* because of the numerical problems. Therefore, the constant-shift method is suggested for a nonlinear transition equation.

SELECTION OF PARAMETERS

When using the constant-shift or the adaptive-shift methods to modify $\mathbf{C}_t$, parameter ε^c has a significant effect on the convergence speed of the DDP procedure. If ε^c is too small the procedure can become divergent because of numerical problems. If ε^c is much larger, the convergence speed is very slow. Generlly, ε^c should be adjusted according to the practical problem. The first derivative of the objective function, with respect to the control variable $\hat{G}_Q(Q_{t,j}, S_{t,j}, t, R_1, R_2)$, has some relation to ε^c. If $\hat{G}_Q(Q_{t,j}, S_{t,j}, t, R_1, R_2)$ is small, ε^c should also be small so that the number of iterations to reach the optimal solution for each time t is as small as possible. In the estuarine-management problem, ε^c is chosen as follows:

$$\begin{aligned} \varepsilon^c &= 0.00001 \quad \text{if } |\hat{G}_Q(Q_{t,j}, S_{t,j}, R_1, R_2, t)| \le 0.1 \\ \varepsilon^c &= 0.0001 \quad \text{if } |\hat{G}_Q(Q_{t,j}, S_{t,j}, R_1, R_2, t)| > 0.1 \end{aligned} \tag{32}$$

The parameter δ in the adaptive-shift method, just like ε^c, has a relation with the first derivative of the objective function with respect to the control variable $\hat{G}_Q(Q_{t,j}, S_{t,j}, R_1, R_2, t)$. For the estuarine-management problem, δ is chosen as

$$\begin{aligned} \delta &= 0.001 \quad \text{if } |\hat{G}_Q(Q_{t,j}, S_{t,j}, R_1, R_2, t)| \le 0.1 \\ \delta &= 0.002 \quad \text{if } |\hat{G}_Q(Q_{t,j}, S_{t,j}, R_1, R_2, t)| > 0.1 \end{aligned} \tag{33}$$

M_1 is the number of iterations of the constant-shift procedure when the adaptive-shift method is used. If M_1 is too small, the control policy may be far from the optimal policy. Therefore, when switching to an active-shift procedure, the convergence speed is very slow. If M_1 is too large, there can be many useless iterations. A suitable M_1 should be a value such that the total number of iterations is as small as possible. $M_1 = 10$ is suggested for the estuarine-management problem.

It is known that when penalty parameters approach infinity, the penalty terms approach zero. However, for practical problems, it may not be necessary to make penalty parameters go to infinity. The values of R_1 and R_2 should be such that the penalty terms on state variables and control variables have the same level of effect on the objective function. In the estuarine-management problem, the control variable, freshwater inflow, has the order of 10^4 and the state variables have the order 10^1. Therefore, R_2 should have a larger value to force these two penalty terms to reach the same level of penalty as the objective function. The values of $R_1 = 10$ and $R_2 = 10{,}000$ were used. R_1 and R_2 are updated according to the following method:

$$R_1^{k+1} = \Delta R_1 \cdot R_1^k; \qquad R_2^{k+1} = \Delta R_2 \cdot R_2^k \tag{34, 35}$$

where k = number of iterations and

$$\Delta R_1 = 10 \quad \text{when } \max_{t,j}[\max(|Q_{t,j} - \underline{Q}'_{t,j}|, |\bar{Q}'_{t,j} - Q_{t,j}|)] \ge \varepsilon_Q \tag{36a}$$

$$\Delta R_1 = 1 \quad \text{when } \max_{t,j}[\max(|Q_{t,j} - \underline{Q}'_{t,j}|, |\bar{Q}'_{t,j} - Q_{t,j}|)] < \varepsilon_Q \tag{36b}$$

$$\Delta R_2 = 10 \quad \text{when } \max_{t,j}[\max(|S_{t,j} - \underline{S}'_{t,j}|, |\bar{S}'_{t,j} - S_{t,j}|)] \ge \varepsilon_S \tag{37a}$$

$$\Delta R_2 = 1 \quad \text{when } \max_{t,j}[\max(|S_{t,j} - \underline{S}'_{t,j}|, |\bar{S}'_{t,j} - S_{t,j}|)] < \varepsilon_S \tag{37b}$$

where ε_Q and ε_S = given bound violation criteria for freshwater inflow and salinity, respectively.

APPLICATION

The DDP method for linear and nonlinear transition equations is applied to the Lavaca-Tres Palacios Estuary in Texas. The optimal monthly freshwater inflows and fishery harvest for linear and nonlinear transition equations are listed in Table 7. Results indicate that for the Lavaca River, the monthly freshwater inflows in January, February, March, July, August, November, and December are small in many cases and are at the lower bounds. This is because the coefficients of freshwater inflow in the fishery harvest equation (for all shellfish) are negative values. For the Colorado River, the freshwater inflow in January, February, March, November, and December are relatively small.

TABLE 7. Optimal Freshwater Inflows and Fishery Harvest for Different Transition Equations

	Linear Transition Equation		Nonlinear Transition Equation	
Month (1)	Freshwater inflow from Lavaca River (cfs) (2)	Freshwater inflow from Colorado River (cfs) (3)	Freshwater inflow from Lavaca River (cfs) (4)	Freshwater inflow from Colorado River (cfs) (5)
January	37	293	37	424
February	1,112	324	41	1,934
March	1,283	293	37	837
April	3,784	6,070	2,442	1,619
May	2,256	2,429	2,172	3,179
June	2,257	2,429	2,172	3,180
July	656	6,017	114	3,308
August	732	2,900	114	2,900
September	3,272	653	2,184	1,966
October	2,569	2,429	2,171	3,179
November	114	573	114	1,087
December	1,065	496	111	844
Harvest (1,000 lb)	6,693		7,294	

Note: 1 cfs = 0.0283 m³/s, 1 lb = 0.4536 kg.

TABLE 8. Optimal Fishery Harvests for Different Transition Equations and Different Initial Values (Using Bracket Penalty Function)

Transition equations (1)	Cases (2)	Initial values (cfs) (3)	Optimal fishery harvest (1,000 lb) (4)	Number of iterations (5)
Linear	1	$Q_{initial} = \underline{Q}$	6,693	14
	2	$Q_{initial} = \hat{Q}$	6,693	9
	3	$Q_{initial} = (1/2)(\underline{Q} + \hat{Q})$	6,693	10
Nonlinear	1	$Q_{initial} = \underline{Q}$	7,239	164
	2	$Q_{initial} = \hat{Q}$	7,033	1,830
	3	$Q_{initial} = (1/2)(\underline{Q} + \hat{Q})$	7,033	561

Note: 1 lb = 0.4536 kg; 1 cfs = 0.0283 m³/s.

TABLE 9. Optimal Fishery Harvest for Different Transition Equations and Different Penalty Functions ($Q_{initial} = \underline{Q}$)

Transition equations (1)	Penalty functions (2)	Optimal fishery harvest (1,000 lb) (3)	Number of iterations (4)
Linear	Bracket penalty function	6,693	14
	Lagrangian penalty functions	6,696	29
	Hyperbolic penalty function	6,691	34
Nonlinear	Bracket penalty function	7,239	164
	Lagrangian penalty function	7,246	344
	Hyperbolic penalty function	7,287	322

Note: 1 lb = 0.4536 kg.

TABLE 10. Optimal Monthly Freshwater Inflows and Fishery Harvests for Linear Transition Equations Using Different Methodologies

	DDP METHOD[a]		MATHEMATICAL PROGRAMMING METHOD (USING GAMS)[b]	
	Optimal Freshwater Inflow (cfs)		Optimal Freshwater Inflow (cfs)	
Month (1)	Lavaca River (2)	Colorado River (3)	Lavaca River (4)	Colorado River (5)
January	37	293	37	293
February	1,112	324	1,112	324
March	1,283	293	1,284	293
April	3,784	6,070	3,784	6,070
May	2,256	2,429	2,256	2,429
June	2,257	2,429	2,256	2,429
July	656	6,017	659	6,017
August	732	2,900	732	2,900
September	3,272	653	3,272	653
October	2,569	2,429	2,569	2,429
November	114	573	114	573
December	1,065	496	1,065	496

Note: 1 cfs = 0.0283 m³/s; 1 lb = 0.4536 kg; CPU times are for personal computer MAG486SX33. The optimal fishery harvest under both methods = 6,693 (1,000 lb).
[a]CPU times (in seconds) = 0.73, and number of iterations = 14.
[b]CPU times (in seconds) = 6.12, and number of iterations = 33.

The effects of initial values are indicated by three different initial values (freshwater inflows). In the first case, initial values are equal to the lower bounds. In the second case, initial values are the upper bounds, and in the third case, initial values equal the mean of the lower bounds and the upper bounds. The results of these three cases for different transition equations are listed in Table 8. Results indicate that the optimal fishery harvests are the same for cases 2 and 3, but the case 1 results are different from those of case 2 and case 3 when using the nonlinear transition equation. Because of the nonlinearity of the transition equation, the solutions are not unique. The number of iterations are different for different initial values. The number of iterations for case 1 is the smallest and the number of iterations for case 2 is the largest. For the linear transition equation, the number of iterations reduces dramatically and the solution is unique for different initial values.

To compare the effect of the penalty functions, the Lagrangian penalty function and hyperbolic penalty function are also used. The results for different penalty functions are shown in Table 9. Table 9 indicates that for the nonlinear transition equation, optimal fishery harvests are different for different penalty functions but the differences are small. The bracket penalty function resulted in the smallest number of iterations and the Lagrangian penalty function has the largest number of iterations. For the linear transition equation, the optimal fishery harvests are basically the same for different penalty functions. The bracket penalty function has the smallest number of iterations and the hyperbolic penalty function has the largest. The bracket penalty function seems to work well.

To compare the DDP method with the mathematical programming method, GAMS (Brooke et al. 1988) was also used to solve the problem with the results listed in Tables 10 and 11. For the linear transition equation, the optimal freshwater inflows and fishery harvest are the same using DDP and GAMS. For the nonlinear transition equation, the optimal monthly freshwater inflows are the same for the Lavaca River using DDP and GAMS. The optimal monthly freshwater inflows

TABLE 11. Optimal Monthly Freshwater Inflows and Fishery Harvests for NonLinear Transition Equations Using Different Methodologies

Month (1)	DDP METHOD[a] Optimal Freshwater Inflow (cfs) Lavaca River (2)	Colorado River (3)	MATHEMATICAL PROGRAMMING METHOD (USING GAMS)[b] Optimal Freshwater Inflow (cfs) Lavaca River (4)	Colorado River (5)
January	37	424	37	424
February	41	1,934	41	1,934
March	37	837	37	837
April	2,442	1,619	2,441	4,100
May	2,172	3,179	2,171	3,179
June	2,172	3,180	2,171	3,179
July	114	3,308	114	3,568
August	114	2,900	114	2,085
September	2,184	1,966	2,184	1,718
October	2,171	3,179	2,171	3,179
November	114	1,087	114	1,174
December	111	844	114	750

Note: 1 cfs = 0.0283 m³/s; 1 lb = 0.4536 kg; CPU times are for personal computer MAG486SX33.

[a]Optimal fishery harvest = 7,294 (1,000 lb), CPU times (in seconds) = 3.32, and number of iterations = 164.

[b]Optimal fishery harvest = 7,378 (1,000 lb), CPU times (in seconds) = 7.98, and number of iterations = 280.

for the Colorado River are different for some months for the two approaches. This indicates that for nonlinear problems, the solutions are not unique. The number of iterations using the DDP method is smaller than that using GAMS, which indicates that the DDP method is better than the GAMS method for the estuarine-management model.

Changes in estuarine salinity patterns are a function of several variables, including the magnitude of freshwater inflow, tidal mixing, density currents, wind-induced mixing, evaporation, and salinity. The linear and nonlinear freshwater inflow-salinity relationship can only be used to provide preliminary estimates of the response of the estuary to proposed freshwater inflows. The best salinity patterns can be simulated by the hydrodynamic transport model HYD-SAL (Masch et al. 1971) as used by Bao and Mays (1994). HYD-SAL solves the two-dimensional partial-differential equations of the flux and salinity using the finite-difference method. Using the DDP method presented in this paper, interfaced with HYD-SAL, may be advantageous over using the mathematical programming methodology for the discrete-time optimal control problem proposed by Bao (1992) and Bao and Mays (1994).

CONCLUSIONS

The DDP methodology has been successfully applied to the estuarine-management problem. Four conclusions can be drawn from the development and application of the model.

When the penalty function method is used for the constrained discrete-time optimal control problems, the modified step-search technique can result in the optimal control policy.

Use of the constant-shift and adaptive-shift methods to obtain the positive-definite matrices C_t can guarantee the quadratic convergence of the DDP procedure. Constant ε^c and δ should be values that make the DDP converge faster with no numerical problems.

Different penalty functions have different effects on the convergence speed of the DDP procedure. For the estuarine-management problem, the bracket penalty function works well. The hyperbolic penalty function also makes the DDP procedure converge fast, but this function has several parameters that must be estimated

Compared with the mathematical programming code GAMS, the DDP method converges with fewer iterations and central processing unit (CPU) times.

If the transition equation is a complicated equation, such as the finite-difference equation, using the DDP method should be more advantageous. The next step in this research will be to use the two-dimensional hydrodynamic-salinity transport model HYD-SAL as the simulator to replace the simplified linear or nonlinear transition equations described here. The work in this paper aims to demonstrate the potential of DDP to develop a more complex model that uses a two-dimensional hydrodynamic-salinity transport model (simulator) for the transition.

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